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**1.**

f (*x*) = *x*3 + 2*x*2 – 8*x* + 5

(*a*)Find f′′(*x*)

**(2)**

(*b*)(i) Solve f′′(*x*) = 0

(ii) Hence find the range of values of *x* for which f (*x*) is concave.

**(2)**

**(Total for Question 1 is 4 marks)**

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**2.** A sequence ... is defined by

 = 35



(*a*)(i) Show that  = 40

(ii) Find the value of  and the value of 

**(3)**

Given that the sequence is periodic with order 4

(*b*)(i) write down the value of 

(ii) find the value of **

**(3)**

**(Total for Question 2 is 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**3.** Given that

log2 (*x* + 3) + log2 (*x* + 10) = 2 + 2 log2 *x*

(*a*)show that

3*x*2 – 13*x* – 30 = 0

**(3)**

(*b*)(i) Write down the roots of the equation

3*x*2 – 13*x* – 30 = 0

(ii) Hence state which of the roots in part (*b*)(i) is not a solution of

log2 (*x* + 3) + log2 (*x* + 10) = 2 + 2 log2 *x*

giving a reason for your answer.

**(2)**

**(Total for Question 3 is 5 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**4.** Coffee is poured into a cup.

The temperature of the coffee, *H* °C, *t* minutes after being poured into the cup is

modelled by the equation

*H* = *A*e– *Bt* + 30

where *A* and *B* are constants.

Initially, the temperature of the coffee was 85 °C.

(*a*)State the value of *A*.

**(1)**

Initially, the coffee was cooling at a rate of 7.5 °C per minute.

(*b*)Find a complete equation linking *H* and *t*, giving the value of *B* to 3 decimal places.

**(3)**

**(Total for Question 4 is 4 marks)**

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**5.** The curve *C* has equation *y* = f (*x*)

The curve

• passes through the point *P* (3, –10)

• has a turning point at *P*

Given that



where *k* is a constant,

(*a*)show that *k* = 12

**(2)**

(*b*)Hence find the coordinates of the point where *C* crosses the *y*-axis.

**(3)**

**(Total for Question 5 is 5 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**6.** Relative to a fixed origin *O*,

• *A* is the point with position vector 12**i**

• *B* is the point with position vector 16**j**

• *C* is the point with position vector (50**i** + 136**j**)

• *D* is the point with position vector (22**i** + 24**j**)

(*a*)Show that *AD* is parallel to *BC*.

**(2)**

Points *A*, *B*, *C* and *D* are used to model the vertices of a running track in the shape of

a quadrilateral.

Runners complete one lap by running along all four sides of the track.

The lengths of the sides are measured in metres.

Given that a particular runner takes exactly 5 minutes to complete 2 laps,

(*b*)calculate the average speed of this runner, giving the answer in kilometres per hour.

**(4)**

**(Total for Question 6 is 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**7. In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

A curve has equation

*x*3 + 2*xy* + 3*y*2 = 47

(*a*)Find  in terms of *x* and *y*

**(4)**

The point *P*(–2, 5) lies on the curve.

(*b*)Find the equation of the normal to the curve at *P*, giving your answer in the form

*ax* + *by* + *c* = 0 , where *a*, *b* and *c* are integers to be found.

**(3)**

**(Total for Question 7 is 7 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**8.** (*a*)Express 2cos *θ* + 8sin *θ* in the form *R* cos(*θ* – *α*) , where *R* and *α* are constants,

*R* > 0 and 0 < *α* < 

Give the exact value of *R* and give the value of *α* in radians to 3 decimal places.

**(3)**

The first three terms of an arithmetic sequence are

cos *x* cos *x* + sin *x* cos *x* + 2 sin *x x* ≠ *nπ*

Given that *S*9 represents the sum of the first 9 terms of this sequence as *x* varies,

(*b*)(i) find the exact maximum value of *S*9

(ii) deduce the smallest positive value of *x* at which this maximum value

of *S*9 occurs.

**(3)**

**(Total for Question 8 is 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**9.** The curve *C* has parametric equations

*x* = *t*2 + 6*t* – 16 *y* = 6 ln (*t* + 3) *t* > –3

(*a*)Show that a Cartesian equation for *C* is

*y* = *A* ln (*x* + *B*) *x* > –*B*

where *A* and *B* are integers to be found.

**(3)**

The curve *C* cuts the *y*-axis at the point *P*

(*b*)Show that the equation of the tangent to *C* at *P* can be written in the form

*ax* + *by* = *c* ln 5

where *a*, *b* and *c* are integers to be found.

**(4)**

**(Total for Question 9 is 7 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**10. ** where *k* is a positive constant

(*a*)Express f (*x*) in partial fractions in terms of *k*.

**(3)**

(*b*)Hence find the exact value of *k* for which



**(4)**

**(Total for Question 10 is 7 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**11.**

A diagram of a rectangular object

Description automatically generated with low confidence

A tank in the shape of a cuboid is being filled with water.

The base of the tank measures 20 m by 10 m and the height of the tank is 5 m, as shown

in Figure 1.

At time *t* minutes after water started flowing into the tank the height of the water was

*h* m and the volume of water in the tank was *V* m3

In a model of this situation

• the sides of the tank have negligible thickness

• the rate of change of *V* is inversely proportional to the square root of *h*

(*a*)Show that



where *λ* is a constant.

**(3)**

Given that

• initially the height of the water in the tank was 1.44 m

• exactly 8 minutes after water started flowing into the tank the height of the water

was 3.24 m

(*b*)use the model to find an equation linking *h* with *t*, giving your answer in the form

**

where *A* and *B* are constants to be found.

**(5)**

(*c*)Hence find the time taken, from when water started flowing into the tank, for the

tank to be completely full.

**(2)**

**(Total for Question 11 is 10 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**12.**

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The number of subscribers to two different music streaming companies is

being monitored.

The number of subscribers, *N*A , in thousands, to **company A** is modelled by

the equation

*N*A = |*t* – 3| + 4 *t* ≥ 0

where *t* is the time in years since monitoring began.

The number of subscribers, *N*B , in thousands, to **company B** is modelled by

the equation

*N*B = 8 – |2*t* – 6| *t* ≥ 0

where *t* is the time in years since monitoring began.

Figure 2 shows a sketch of the graph of *N*A and the graph of *N*B over a 5-year period.

**Use the equations of the models to answer parts (a), (b), (*c*)and (d).**

(*a*)Find the initial difference between the number of subscribers to **company A** and the

number of subscribers to **company B**.

**(2)**

When *t* = *T* **company A** reduced its subscription prices and the number of

subscribers increased.

(*b*)Suggest a value for *T*, giving a reason for your answer.

**(2)**

(*c*)Find the range of values of *t* for which *N*A > *N*B giving your answer in set notation.

**(5)**

(*d*)State a limitation of the model used for **company B**.

**(1)**

**(Total for Question 12 is 10 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**13. In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(*a*)Find the first three terms, in ascending powers of *x*, of the binomial expansion of

(3 + *x*)–2

writing each term in simplest form.

**(4)**

(*b*)Using the answer to part (*a*)and using algebraic integration, estimate the value of



giving your answer to 4 significant figures.

**(4)**

(*c*)Find, using algebraic integration, the exact value of



giving your answer in the form *a* ln *b* + *c*, where *a*, *b* and *c* are constants to

be found.

**(5)**

**(Total for Question 13 is 13 marks)**

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**14. In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(*a*)Show that the equation

2tan *θ* (8cos *θ* + 23sin2 *θ*) = 8sin 2*θ* (1 + tan2 *θ*)

may be written as

sin 2*θ* (*A* cos2 *θ* + *B* cos *θ* + *C*) = 0

where *A*, *B* and *C* are constants to be found.

**(3)**

(*b*)Hence, solve for 360° ≤ *x* ≤ 540°

2 tan *x* (8 cos *x* + 23 sin2 *x*) = 8 sin 2*x* (1 + tan2 *x*) *x* ∈ ℝ *x* ≠ 450°

**(4)**

**(Total for Question 14 is 7 marks)**

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**15.** A student attempts to answer the following question:

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The student starts the proof with:

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The start of the student’s proof is reprinted below.

Complete the proof.

**(3)**

Assume that sin *x* – cos *x* < 1 when *x* is an obtuse angle

⟹ (sin *x* – cos *x*)2 < 1

**(Total for Question 15 is 3 marks)**

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**TOTAL FOR PAPER IS 100 MARKS**