**GCE A level Pure Mathematics (9MA0) – Paper 2**

**Pure Mathematics 2**

**October 2021 student-friendly mark scheme**

**Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn’t show follow-through marks (marks that are awarded despite errors being made) or special cases.**

**It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.**

**This document is intended for guidance only and may differ significantly from the final mark scheme published in December 2021.**

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| **Guidance on the use of codes within this document** |
| M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect.  A1 – accuracy mark. This mark is generally given for a correct answer following correct working.  B1 – accuracy mark. This mark is usually given when working and the answer cannot easily be separated.  Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer). |

**Question 1 (Total 4 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) |  | M1 | This mark is given for using correct method to find the common difference. |
|  | A1 | This mark is given for finding the correct value of the common difference. |
| (b) |  | M1 | This mark is given for using the correct formula with *a* = 16, *n* = 500 and their numerical *d* they found in part (a). |
|  | A1 | This mark is given for finding the correct value. |

**Question 2 (Total 5 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) |  | B1 | This mark is given for giving the correct range. |
| (b) |  | M1 | This mark is given for using full method to find f(1.8) and then substitute to find gf(*x*). |
|  | A1 | This mark is given for finding the correct value. |
| (c) |  | M1 | This mark is given for correct attempt to cross multiply and factorising out *x*. |
|  | A1 | This mark is given for correct expression for g-1(*x*). |

**Question 3 (Total 3 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
|  |  | B1 | This mark is given for applying at least one addition or subtraction law of logs correctly. |
|  | M1 | This mark is given for fully correcting working when removing the log and obtain a correct equation and solve for *y*. |
|  | A1 | This mark is given for the correct exact value. |

**Question 4 (Total 3 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| **4** |  | M1 | This mark is given for attempting to use at least one correct approximation. |
|  | M1 | This mark is given for attempting to use correct approximations to obtain an expression in terms of *θ* only. |
|  | A1 | This mark is given for all correct terms following correct work. |

**Question 5 (Total 7 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| **(a)(i)** |  | M1 | This mark is given for reducing the power by 1 for at least one of the terms |
|  | A1 | This mark is given for correct expression of |
| **(ii)** |  | A1 | This mark is given for correct expression of |
| **(b)(i)** |  | M1 | This mark is given for substituting  *x* = 1 into |
| Therefore there is a stationary point  at *x* = 1 | A1 | This mark is given for obtaining and conclude there is a stationary point at *x* = 1 |
| (b)(ii) |  | M1 | This mark is given for calculating the second derivative either side of *x =* 1. |
| Hence point of inflection | A1 | This mark is given for calculating correct  with a reasoned conclusion stating that the stationary point is a point of inflection. |

**Question 6 (Total 5 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| **(a)** | Angle *AOB* = | B1 | This mark is given for deducing the correct expression for angle *AOB*. |
| **(b)** | Area = | M1 | This mark is given for using correct method for finding the area using the angle from part (a). |
|  | A1 | This mark is given for the correct proof. |
| **(c)** | Perimeter = | M1 | This mark is given for using the correct method to fins the perimeter. |
| or | A1 | This mark is given for finding the perimeter simplified. |

**Question 7 (Total 9 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| **(a)** |  | B1 | This mark is given for finding correct |
|  | M1 | This mark is given for substituting  *x* = 5 into |
|  | M1 | This mark is given for using correct method to find the equation of *l*. |
|  | A1 | This mark is given for correct solution of equation of *l*. |
| **(b)** | Both *C* and *l* pass through (0, −23)  and so *C* meets *l* again on the *y*-axis | B1 | This mark is given for provide suitable deduction. |
| **(c)** |  | M1 | This mark is given for attempting to integrate using the correct method. |
| A1 | This mark is given for finding correct integration. |
|  | M1 | This mark is given for using the correct method to find the area, by using the correct limits. |
|  | A1 | This mark is given for calculating the exact value. |

**Question 8 (Total 9 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| **(a)** | , | M1 | This mark is given for using an appropriate method of differentiation. |
|  | A1 | This mark is given for fully correct differentiation. |
|  | M1 | This mark is given for a valid attempt to make  the subject. |
| A1 | This mark is given for the fully correct expression of . |
| **(b)** |  | M1 | This mark is given for Uses  and in the equation of C to obtain an equation in p and q. |
|  | B1 | This mark is given for finding the correct gradient of the given normal. |
|  | M1 | This mark is given for correct strategy to establish an equation connecting *p* and *q* using *x* = −1 and *y* = −4 in and the gradient of the normal. |
|  | M1 | This mark is given for solving the simultaneous equations to find the value of *p* and *q*. |
|  | A1 | This mark is given for correct values of *p* and *q*. |

**Question 9 (Total 3 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
|  |  | B1 | This mark is given for finding the first term or the common ratio. |
|  | M1 | This mark is given for recognising this is a infinite geometric series and applying the sum to infinity formula with *a* and *r*. |
|  | A1 | This mark is given for the correct proof. |

**Question 10 (Total 6 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| **(a)** |  | M1 | This mark is given for taking logs on both sides and showing the addition law. |
|  | A1 | This mark is given for using the power law to obtain the equation given with no errors. |
| **(b)** |  | B1 | This mark is given for deducing the correct value of *b*. |
|  | M1 | This mark is given for substituting one of the given points and the value of *b* into the equation found in part (a). |
|  | A1 | This mark is given for the complete equation of *T*. |
| **(c)** | The time taken for one swing of a pendulum of length 1 m | B1 | This mark is given for correct interpretation. |

**Question 11 (Total 10 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| **(a)** | −2*k*  (1.5*k*, *k*)  *k*  2*k*  *O* | B1 | This mark is given for shape in any position |
| B1 | This mark is given for correct *x*-intercepts or coordinates |
| B1 | This mark is given for correct  *y*-intercept or coordinates |
| B1 | This mark is given for correct coordinates for the vertex of a shape |
| **(b)** |  | B1 | This mark is given for deducing the correct critical value of . |
|  | M1 | This mark is given for attempting to solve  to find the other critical value. |
|  | A1 | This mark is given for the correct value. |
|  | A1 | This mark is given for correct answer using the correct set notation. |
| **(c)** | *x* = 3*k*  *y* = 3 − 5*k* | B1 | This mark is given for one of the correct coordinates. |
| B1 | This mark is given for both sets of correct coordinates. |

**Question 12 (Total 7 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| **(a)** |  | B1 | This mark is given for correct expression for |
|  | M1 | This mark is given for complete method using given substitution. |
|  | A1 | This mark is given for correct working with correct limits. |
| **(b)** |  | M1 | This mark is given for expanding the ()3 and divide through by *u*, and making progress with the integration. |
|  | A1 | This mark is given for the correct integration. |
|  | M1 | This mark is given for completing the process and apply the ‘changed’ limits and subtract the right way round. |
|  | A1 | This mark is given for fully correct solution. |

**Question 13 (Total 6 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| **(a)** |  | B1 | This mark is given for the correct expression for |
|  | M1 | This mark is given for attempting |
|  | A1 | This mark is given for the correct expression. |
| **(b)** |  | M1 | This mark is given for recognising the need to find the value of when  *y* = 8 |
|  | M1 | This mark is given for attempting to find  using the value of  or *θ.* |
|  | A1 | This mark is given for the exact value of the gradient. |

**Question 14 (Total 12 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| **(a)** |  | B1 | This mark is given for identifying the correct expression for |
|  | B1 | This mark is given for identifying the correct expression for . |
|  | M1 | This mark is given for applying |
|  | A1 | This mark is given for obtaining the correct equation with no errors. |
| **(b)** |  | M1 | This mark is given for separating the variables correctly. |
|  | A1 | This mark is given for the correct equation with no errors. |
|  | M1 | This mark is given for using a correct strategy by separating variable correctly. |
|  | A1 | This mark is given for correct equation. |
|  | M1 | This mark is given for using the correct log work to obtain *h* in terms of *t*. |
|  | A1 | This mark is given for the correct equation. |
| **(c)** | When *h* > 4.8, | M1 | This mark is given for a correct solution. |
| The limit for *h* (according to the model) is 4.8m and the tank is 5m high so the tank will never become full | A1 | This mark is given for correct interpretation for the method |

**Question 15 (Total 11 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| **(a)** |  | B1 | This mark is given for correct value of R. |
|  | M1 | This mark is given for proceeding to a value of . |
|  | A1 | This mark is given for correct value of . |
| **(b)(i)** |  | B1 | This mark is given for the correct maximum height of *P* above the water level. |
| **(ii)** |  | M1 | This mark is given for using to obtain a value for *t* |
| *t* = 11.6 | A1 | This mark is given for correct value. |
| **(c)** |  | M1 | This mark is given for using the model and set  and proceed to |
|  | M1 | This mark is given for solve  to obtain at least one value for *t*. |
| So the time required is | M1 | This mark is given for fully correct method to find the required duration. |
| = 3.34 | A1 | This mark is given for the correct value. |
| **(d)** | the 3 would need to vary | B1 | This mark is given for a correct refinement. |