**GCE AS Further Mathematics (8FM0-21) – Paper 21**

**Further Pure Mathematics**

**Summer 2018 student-friendly mark scheme**

**Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn’t show follow-through marks (marks that are awarded despite errors being made) or special cases.**

**It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.**

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| **Guidance on the use of codes within this document** |
| M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect.  A1 – accuracy mark. This mark is generally given for a correct answer following correct working.  B1 – working mark. This mark is usually given when working and the answer cannot easily be separated.  Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer). |

**Question 1 (Total 7 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | 5 sin *x* + 12 cos *x* = 5+ 12 | M1 | This mark is given for using the correct formulae to express 5 sin *x* + 12 cos *x* in terms of *t* |
| 5+ 12 = 2  5(2*t*) + 12(1 – *t* 2) = 2(1 + *t* 2) | M1 | This mark is given for equating to 2 and removing the fractions |
| 10*t* + 12 – 12*t* 2 = 2 + 2*t* 2  –14*t* 2 + 10*t* + 10 = 0  7*t* 2 – 5*t* – 5 = 0 | A1 | This mark is given for rearranging to find the given result. |
| (b) | *t* = | M1 | This mark is given for a method to solve the quadratic equation 7*t* 2 – 5*t* – 5 = 0 |
| *t* = tan  =  = arctan 1.275 and –0.56 | M1 | This mark is given for a method to find values for |
| = 51.9 and –29.26  *x* = 103.8° and –58.5° | A1 | This mark is given for one solution found |
| A1 | This mark is given for both solutions found |

**Question 2 (Total 7 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | Two iterations for *t* = 0 to *t* = 3  *h* =  = 1.5 | B1 | This mark is given for finding the correct step length *h* |
| *θ* 0 = 80 ⇒ = –0.1(80 – 20) = –6 | M1 | This mark is given for finding the initial value of the differential equation using the initial conditions |
| = –0.6 ⇒ *θ* 1 = 80 – (6 × 1.5) | M1 | This mark is given for a correct application of the first iteration |
| *θ* 1 = 71 | A1 | This mark is given for the correct answer only |
| *θ* 1 = 71 ⇒ = –0.1(71 – 20) = –5.1 | M1 | This mark is given for a correct application of the first iteration |
| *θ* 2 = 71 – (1.5 × 5.1) = 63.35 °C | A1 | This mark is given for the correct answer only |
| (b) | The value of *k* would need to be decreased | B1 | This mark is given for a correct statement |

**Question 3 (Total 7 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | | **Notes** | |
|  | ≤ 0 | M1 | | This mark is given for gathering terms on one side over a common denominator | |
| ≤ 0 | M1 | | This mark is given for simplifying the numerator | |
| A1 | | This mark is given for the correct answer only | |
| Critical values 3, –1 and –3 | B1 | | This mark is given for identifying the correct critical values for the denominator | |
| Critical value – | A1 | | This mark is given for identifying the correct critical value for the numerator | |
| {*x* ∈ ℝ: –3 < *x* < –1} ∪ { *x* ∈ ℝ: – < *x* < –1} | | M1 | | This mark is given for one correct inequality |
| A1 | | This mark is given for the correct answer only |

**Question 4 (Total 9 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | | **Notes** | |
| (a) | = , = | M1 | | This mark is given for a method to find any two edges of the required triangle | |
| Area = ⎜ ×  ⎜ = | M1 | | This mark is given for a method to find an appropriate vector product to find the area | |
| =  =  = 52.2 | A1 | | This mark is given for the correct method to show the area of the triangle | |
| (b) | = , = , = | M1 | | This mark is given for finding appropriate vectors to find the volume of the tetrahedron | |
| *V* =  ×  ×  = | M1 | | This mark is given for a method to use the scalar triple product using suitable vectors | |
| = (–2 × (–20 + 12) – 11(35 + 60) – 2(14 + 40)) | | A1 | | This mark is given for finding a correct numerical expression for the volume of the tetrahedron |
| =  = 189.5 (mm2) | A1 | | This mark is given for the correct answer only | |
| Density =  × 1000 (g cm–3) | M1 | | This mark is given for a method to find the density of the tetrahedron (including a correction for units) | |
| = 2.64 (g cm–3) | A1 | | This mark is given for the correct answer only | |

**Question 5 (Total 10 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | *xy* = *c*2  ⇒ *y* = *c*2*x*–1 ⇒  = –*c*2*x*–2 = –*c*2(*cp*)–2 | M1 | This mark is given for method to find the gradient of the curve at *P* |
| = – so gradient of the normal = *p*2 | A1 | This mark is given for deducing the gradient of the normal given the gradient of the tangent |
| *y* –  = *p*2(*x* – *cp*) | M1 | This mark is given for applying the normal gradient *m* in terms of *p* |
| *py* – *c* = *p*3(*x* – *cp*)  *p*3*x* – *py* + *c*(1 – *p*4) = 0 | A1 | This mark is given for multiplying both sides by *p* and rearranging to find the given result |
| (b) | *xy* = *c*2 and *p*3*x* – *py* + *c*(1 – *p*4) = 0  ⇒ *p*3*x* – *p*  + *c*(1 – *p*4) = 0 | M1 | This mark is given for a method to find a quadratic equation in *x* |
| *p*3*x*2 – *pc* 2 + *cx*(1 – *p*4) = 0 | A1 | This mark is given for finding a correct quadratic equation in *x* |
| (*x* – *cp*)(*p*3*x* + *c*) = 0 | M1 | This mark is given for recognising that (*x* – *cp*) is a factor of the quadratic and using this to find the other quadratic |
| *Q* = | A1 | This mark is given for finding the correct coordinates of the point *Q* |
| Midpoint of *PQ* is | M1 | This mark is given for a method to find the midpoint of the points *P* and *Q* |
| A1 | This mark is given for the correct answer only |