**Paper 2 Option 2A**

**Further Pure Mathematics 1 Mark Scheme (Section A)**

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **1(a)** |  | M1 | 2.1 |
|  | M1 | 1.1b |
|  | A1\* | 2.1 |
|  | **(3)** |  |
| **(b)** |  | M1 | 1.1a |
|  | M1 | 1.1b |
|  | A1\* | 2.1 |
|  | **(3)** |  |
| **(6 marks)** | | | |
| **Notes:** | | | |
| **(a)**  **M1:**  and the *t*-substitutions for both cos *x* and tan *x* to obtain an expression  in terms of *t*  **M1:** Sorts out the sec *x* term, and puts over a common denominator of  **A1\*:** Factorises both numerator and denominator (must be seen) and cancels the  term to  achieve the answer | | | |
| **(b)**  **M1:** Uses the *t*-substitution for sin *x* in both numerator and denominator  **M1:** Multiples through by in numerator and denominator  **A1\*:** Factorises both numerator and denominator and makes the connection with part (a) to  achieve the given result | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **2** | £300 purchased one hour after opening  and  half an hour after purchase  so step *h* required is 0.25 | B1 | 3.3 |
|  | M1 | 3.4 |
|  | M1 | 1.1b |
|  | A1ft | 1.1b |
|  | M1 | 1.1b |
| (nearest £) | A1 | 3.2a |
|  | **(6)** |  |
| **(6 marks)** | | | |
| **Notes:** | | | |
| **B1:** Identifies the correct initial conditions and requirement for *h*  **M1:** Uses the model to evaluate at  using their  and  **M1:** Applies the approximation formula with their values  **A1ft:** 3.5 or exact equivalent. Follow through their step value  **M1:** Attempt to find with their 3.5  **A1:** Applies the approximation and interprets the result to give £396 | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **3** |  |  |  |
|  | M1 | 2.1 |
|  | M1 | 1.1b |
| At least two correct critical values from | A1 | 1.1b |
| All four correct critical values | A1 | 1.1b |
|  | M1  A1 | 2.2a  2.5 |
|  | **(6)** |  |
| **(6 marks)** | | | |
| **Notes:** | | | |
| **M1:** Gathers terms on one side and puts over common denominator, or multiply by  and then gather terms on one side  **M1:** Factorise numerator or find roots of numerator or factorise resulting in equation into 4  factors  **A1:** At least 2 correct critical values found  **A1:** Exactly 4 correct critical values  **M1:** Deduces that the 2 “outsides” and the “middle interval” are required. May be by sketch,  number line or any other means  **A1:** Exactly 3 correct intervals, accept equivalent set notations, but must be given as a set  e.g. accept  or | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **4(a)** | Identifies glued face is triangle *ABC* and attempts to find the area, e.g. evidences by use of | M1 | 3.1a |
|  | M1 | 1.1b |
|  | M1 | 1.1b |
|  | A1 | 1.1b |
|  | **(4)** |  |
| **Alternative** |  | |
| Identifies glued face is triangle *ABC* and attempts to find the area, e.g. evidences by use of | M1 | 3.1a |
| and | M1 | 1.1b |
| So area of glue is = | M1 | 1.1b |
|  | A1 | 1.1b |
|  | **(4)** |  |
| **(b)** | Volume of parallelepiped taken up by concrete is e.g. | M1 | 3.1a |
|  | M1 | 1.1b |
|  | A1 | 1.1b |
| Volume of parallelepiped is 6  volume of tetrahedron (= 10),  so volume of glass is difference between these, viz. | M1 | 3.1a |
| Volume of glass | A1 | 1.1b |
|  | **(5)** |  |

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| **Question** | **Scheme** | **Marks** | **AOs** |
|  | **4(b) Alternative** |  | |
| is perpendicular to both | M1 | 3.1a |
|  | A1 | 1.1b |
| and so height of tetrahedron is | M1 | 3.1a |
| Volume of glass is *V* = 5  Volume of tetrahedron | M1 | 1.1b |
|  | A1 | 1.1b |
|  | **(5)** |  |
| **(c)** | The glued surfaces may distort the shapes / reduce the volume of concrete  Measurements in m may not be accurate  The surface of the concrete tetrahedron may not be smooth  Pockets of air may form when the concrete is being poured | B1 | 3.2b |
|  | **(1)** |  |
| **(10 marks)** | | | |
| **Question 4 notes:** | | | |
| Accept use of column vectors throughout  **(a)**  **M1:** Shows an understanding of what is required via an attempt at finding the area of triangle  *ABC*  **M1:** Any correct method for the triangle area is fine  **M1:** Finds and  or any other appropriate pair of vectors to use in the vector product  and attempts to use them  **A1:** Correct procedure for the vector product with at least 1 correct term or exact  equivalent | | | |
| 1. **Alternative**   **M1:** Finds two appropriate sides and attempts the scalar product and magnitudes of two of the  sides  **M1:** May use different sides to those shown  **M1:** Correct full method to find the area of the triangle using their two sides  **A1:**  or exact equivalent | | | |

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| **Question 4 notes continued:** |
| **(b)**  **M1:** Attempts volume of concrete by finding volume of tetrahedron with appropriate method  **M1:** Uses the formula with correct set of vectors substituted (may not be the ones shown) and  vector product attempted  **A1:** Correct value for the volume of concrete  **M1:** Attempt to find total volume of glass by multiplying their volume of concrete by 6 and  subtracting their volume of concrete. May restart to find the volume of parallelepiped  **A1:**  only, ignore reference to units |
| 1. **Alternative**   **M1:** Notes (or works out using scalar products) that  is a vector perpendicular to both    **A1:** Finds (using that **OA** and **OB** are perpendicular), area of *AOB*  **M1:** Solves  to get the height of the tetrahedron    **M1:** Identifies the correct area as 5 times the volume of the tetrahedron (may be done as in  main scheme via the difference)  **A1:**  only, ignore reference to units |
| **(c)**  **B1:** Any acceptable reason in context |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **5(a)** | and  is a general point on *C* | B1 | 2.2a |
|  | **(1)** |  |
| **(b)** | gives  or  so | M1 | 2.2a |
|  | M1 | 1.1b |
| leading to  \* | A1\* | 2.1 |
|  | **(3)** |  |
| **(c)** | into *l* | M1 | 3.1a |
|  | M1 | 1.1b |
| and  cuts *x*-axis when | M1 | 2.1 |
|  | A1 | 1.1b |
|  | M1 | 2.1 |
|  | M1 | 1.1b |
| A1 | 1.1b |
|  | A1\* | 1.1b |
|  | **(8)** |  |

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| **Question** | | **Scheme** | **Marks** | **AOs** |
|  | | **5(c) Alternative 1** |  | |
| into *l* | M1 | 3.1a |
|  | M1 | 1.1b |
| into *l* gives | M1 | 2.1 |
|  | A1 | 1.1b |
|  | M1 | 2.1 |
|  | M1 | 1.1b |
| A1 | 1.1b |
|  | A1\* | 1.1b |
|  | **(8)** |  |
| **5(c) Alternative 2** |  | |
| into *l* | M1 | 3.1a |
|  | M1 | 1.1b |
| and  cuts p*x*-axis when | M1 | 2.1 |
|  | A1 | 1.1b |
| and  in  gives | M1 | 2.1 |
|  | M1 | 1.1b |
| A1 | 1.1b |
|  | A1\* | 1.1b |
|  | **(8)** |  |
| **(12 marks)** | | | | |
| **Question 5 notes:** | | | | |
| **(a)**  **B1:** Substitutes  into  to obtain  and substitutes  into  to  obtain and concludes that *P* lies on *C* | | | | |
| **(b)**  **M1:** Uses the given formula to deduce the derivative. Alternatively, may differentiate using  chain rule to deduce it  **M1:** Applies with their tangent gradient *m*, which is in terms of *p.*  Accept use of  with a clear attempt to find *c*  **A1\*:** Obtains  by **cso** | | | | |
| **(c)**  **M1:** Substitutes their  and  into *l*  **M1:** Obtains a 3 term quadratic and solves (using the usual rules) to give  **M1:** Substitutes their  (which must be positive) and  into *l* and solves to give  **A1:** Finds that *l* cuts the *x*-axis at  **M1:** Fully correct method for finding the area of *R*  i.e.  **M1:** Integrates  to give where  **A1:** Integrates  to give simplified or un-simplified  **A1\*:** Fully correct proof leading to a correct answer of 36 | | | | |
| **(c) Alternative 1**  **M1:** Substitutes their  and  into *l*  **M1:** Obtains a 3 term quadratic and solves (using the usual rules) to give  Substitutes their  (which must be positive) into *l* and rearranges to give  **M1:** Finds *l* as  **A1:** Fully correct method for finding the area of *R*  **M1:** i.e.  **M1:** Integrates  to give where  **A1:** Integrates to give  simplified or un-simplified  **A1\*:** Fully correct proof leading to a correct answer of 36 | | | | |

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| **Question 5 notes continued:** | |
| **(c) Alternative 2**  **M1:** Substitutes their  and  into *l*  **M1:** Obtains a 3 term quadratic and solves (using the usual rules) to give  **M1:** Substitutes their  (which must be positive) and  into *l* and solves to give  **A1:** Finds that *l* cuts the *x*-axis at  **M1:** Fully correct method for finding the area of *R*  i.e.  **M1:** Integrates  to give where  **A1:** Integrates  to give simplified or un-simplified  **A1\*:** Fully correct proof leading to a correct answer of 36 |

**Further Pure Mathematics 2 Mark Scheme (Section B)**

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **6(a)** | Consider | M1 | 1.1b |
| So  is characteristic equation | A1 | 1.1b |
|  | **(2)** |  |
| So | B1ft | 1.1b |
| **(b)** | Multiplies both sides of their equation by **A** so | M1 | 3.1a |
| Uses  So \* | A1\*cso | 1.1b |
|  | **(3)** |  |
| **(5 marks)** | | | |
| **Notes:** | | | |
| **(a)**  **M1:** Complete method to find characteristic equation  **A1:** Obtains a correct three term quadratic equation – may use variable other than | | | |
| **(b)**  **B1ft:** Uses Cayley Hamilton Theorem to produce equation replacing  with **A** and constant term with constant multiple of identity matrix, **I**  **M1:** Multiplies equation by **A**  **A1\*:** Replaces by linear expression in **A** and achieves printed answer with no errors | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **7(i)** | Adding digits 8 + 1 + 8 + 4 = 21 which is divisible by 3 ( or continues to add digits giving 2+1=3 which is divisible by 3 ) so concludes that 8184 is divisible by 3 | M1 | 1.1b |
| 8184 is even, so is divisible by 2 and as divisible by both 3 and 2, so it is divisible by 6 | A1 | 1.1b |
|  | **(2)** |  |
| **(ii)** | Starts Euclidean algorithm 31=27 ×1 + 4 and 27 = 4 × 6 + 3 | M1 | 1.2 |
| 4 = 3 × 1 + 1 ( so hcf = 1) | A1 | 1.1b |
| So 1 = 4 – 3 × 1 = 4 – (27 – 4 × 6) × 1 = 4 × 7 – 27 × 1 | M1 | 1.1b |
| (31 – 27 × 1) × 7 – 27 ×1 = 31 × 7 – 27 × 8  *a* = –8 and *b* = 7 | A1cso | 1.1b |
|  | **(4)** |  |
| **(6 marks)** | | | |
| **Notes:** | | | |
| **(i)**  **M1:** Explains divisibility by 3 rule in context of this number by adding digits  **A1:** Explains divisibility by 2, giving last digit even as reason and makesconclusion that  number is divisible by 6 | | | |
| **(ii)**  **M1:** Uses Euclidean algorithm showing two stages  **A1:** Completes the algorithm. Does not need to state that hcf = 1  **M1:** Starts reversal process, doing two stages and simplifying  **A1cso:** Correct completion, giving clear answer following complete solution | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **8(a)** |  | M1 | 2.1 |
| which is the equation of a circle | A1\* | 2.2a |
| As so | M1 | 1.1b |
| Giving centre at (-3, 4) and radius = 10 | A1ft | 1.1b |
|  | **(4)** |  |
| **(b)** |  | M1 | 1.1b |
| A1 | 1.1b |
|  | **(2)** |  |
| **(c)** | Values range from **their** – 3 –10 to their –3 + 10 | M1 | 3.1a |
| So | A1ft | 1.1b |
|  | **(2)** |  |
| **(8 marks)** | | | |
| **Notes:** | | | |
| **(a)**  **M1:** Obtains an equation in terms of *x* and *y* using the given information  **A1:** Expands and simplifies the algebra, collecting terms and obtains a circle equation  correctly, deducing that this is a circle  **M1:** Completes the square for their equation to find centre and radius  **A1ft:** Both correct | | | |
| **(b)**  **M1:** Draws a circle with centre and radius as given from **their** equation  **A1:** Correct circle drawn, as above, with centre at 3 + 4i and passing through all four  quadrants | | | |
| **(c)**  **M1:** Attempts to find where a line parallel to the real axis, passing through the centre of the  circle, meets the circle so using “ their – 3 –10” to “their –3 + 10”  **A1ft:** Correctly obtains the correct answer for their centre and radius | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **9(a)(i)** | |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | \* | 0 | 2 | 3 | 4 | 5 | 6 | | 0 | 0 | 2 | 3 | 4 | 5 | 6 | | 2 | 2 | 0 |  |  | 4 |  | | 3 | 3 |  |  |  |  | 5 | | 4 | 4 |  |  |  |  |  | | 5 | 5 | 4 |  |  |  |  | | 6 | 6 |  | 5 |  |  |  | | M1 | 1.1b |
| |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | \* | 0 | 2 | 3 | 4 | 5 | 6 | | 0 | 0 | 2 | 3 | 4 | 5 | 6 | | 2 | 2 | 0 | 6 | 5 | 4 | 3 | | 3 | 3 | 6 | 4 | 2 | 0 | 5 | | 4 | 4 | 5 | 2 | 6 | 3 | 0 | | 5 | 5 | 4 | 0 | 3 | 6 | 2 | | 6 | 6 | 3 | 5 | 0 | 2 | 4 | | M1  A1 | 1.1b  1.1b |
| **(ii)** | Identity is zero and there is closure as shown above | M1 | 2.1 |
| 3 and 5 are inverses, 4 and 6 are inverses, 2 is self-inverse,  0 is identity so is self-inverse | M1 | 2.5 |
| Associative law may be assumed so *S* forms a group | A1 | 1.1b |
|  | **(6)** |  |
| **(b)** | 4\*4\*4 = 4\* (4 \* 4) = 4 \* 6 or 4\*4\*4 = (4\* 4) \* 4 = 6\* 4 | M1 | 2.1 |
| = 0 (the identity) so 4 has order 3 | A1 | 2.2a |
|  | **(2)** |  |
| **(c)** | 3 and 5 each have order 6 so either generates the group | M1 | 3.1a |
| **Either**  **Or** | A1, A1 | 1.1b  1.1b |
|  | **(3)** |  |
| **(11 marks)** | | | |

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| **Question 9 notes:** |
| **(a)(i)**  **M1:** Begins completing the table – obtaining correct first row and first column and using  symmetry  **M1:** Mostly correct – three rows or three columns correct (so demonstrates understanding of  using \*  **A1:** Completely correct |
| **(a)(ii)**  **M1:** States closure and identifies the identity as zero  **M1:** Finds inverses for each element  **A1:** States that associative law is satisfied and so all axioms satisfied and *S* is a group |
| **(b)**  **M1:** Clearly begins process to find 4\*4\*4 reaching 6\*4 or 4\*6 with clear explanation  **A1:** Gives answer as zero, states identity and deduces that order is 3 |
| **(c)**  **M1:** Finds either 3 or 5 or both  **A1:** Expresses four of the six terms as powers of either generator correctly (may omit identity  and generator itself)  **A1:** Expresses all six terms correctly in terms of either 3 or 5 (Do not need to give both) |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **10(a)** | is the population at the end of year *n* – 1 and this is increased by 10% by the end of year *n*, so is multiplied by 110% = 1.1 to give  as new population by natural causes | B1 | 3.3 |
| *Q* is subtracted from as *Q* is the number of deer removed from the estate | B1 | 3.4 |
| So  as population at start is 5000 and | B1 | 1.1b |
|  | **(3)** |  |
| **(b)** | Let *n* = 0, then  so result is true when *n* = 0 | B1 | 2.1 |
| Assume result is true for *n* = *k* ,  then **as**  so | M1 | 2.4 |
|  | A1 | 1.1b |
| So | A1 | 1.1b |
| Implies result holds for *n* = *k* + 1 and so by induction is true for all integer *n* | B1 | 2.2a |
|  | **(5)** |  |
| **(c)** | For *Q* < 500 the population of deer will grow, for *Q* > 500 the population of deer will fall | B1 | 3.4 |
| For *Q* = 500 the population of deer remains steady at 5000, | B1 | 3.4 |
|  | **(2)** |  |
| **(10 marks)** | | | |
| **Notes:** | | | |
| **(a)**  **B1:** Need to see 10% increase linked to multiplication by scale factor 1.1  **B1:** Needs to explain that subtraction of *Q* indicates the removal of *Q* deer from population  **B1:** Needs complete explanation with mention of being the  initial number of deer | | | |
| **(b)**  **B1:** Begins proof by induction by considering *n* = 0  **M1:** Assumes result is true for *n* = *k* and uses iterative formula to consider *n* = *k* + 1  **A1:** Correct algebraic statement  **A1:** Correct statement for *k* + 1 in required form  **B1:** Completes the inductive argument | | | |
| **(c)**  **B1:** Consideration of both possible ranges of values for *Q* as listed in the scheme  **B1:** Gives the condition for the steady state | | | |