

9B Differentiating Exponentials

e^x

$\ln(x)$

$$a^x$$

1. Find $\frac{dy}{dx}$ given that:

$$y = e^{3x} + 2^{3x}$$

2. Find $\frac{dy}{dx}$ given that:

$$y = \ln(x^3) + \ln 7x$$

3. Find $\frac{dy}{dx}$ given that:

$$y = \frac{2 - 3e^{7x}}{4e^{3x}}$$

9C (Part 2) The Chain Rule

1. Given that $y = (3x^4 + x)^5$, find $\frac{dy}{dx}$ using the chain rule.

2. Given that $y = \sin^4 x$, find $\frac{dy}{dx}$

3. Given that $y = \sqrt{5x^2 + 1}$, find the gradient of the curve at (4,9).

9D The Product Rule

1. Given that $f(x) = x^2\sqrt{3x-1}$, find $f'(x)$.

2. Given that $y = e^{4x} \sin^2 3x$, show that $\frac{dy}{dx} = e^{4x} \sin 3x (A \cos 3x + B \sin 3x)$, where A and B are constants to be found.

9E The Quotient Rule

1. Given that $y = \frac{x}{2x+5}$, find $\frac{dy}{dx}$

2. A curve C has equation $y = \frac{\sin x}{e^{2x}}$, $0 < x < \pi$.

The curve has a stationary point at P . Find the coordinates of P , to 3 significant figures.

9A Differentiating Sine & Cosine

Small angle notes

Differentiating Sine from First Principles

Differentiating $\sin(kx)$ from First Principles

1. Find $\frac{dy}{dx}$ given that:

a)

$$y = \sin 2x$$

b)

$$y = 3\sin 2x$$

c)

$$y = 3\cos x + 2\sin 4x$$

2. A curve has equation:

$$y = \frac{1}{2}x - \cos 2x$$

Find the stationary points on the curve in the interval $0 \leq x \leq \pi$.

9F Trigonometric Derivatives

1. Given that $y = \tan x$, find $\frac{dy}{dx}$

2. Given that $y = \operatorname{cosec} x$, find $\frac{dy}{dx}$.

3. Given that $y = \sec x$, find $\frac{dy}{dx}$.

4. Given that $y = \cot x$, find $\frac{dy}{dx}$.

5. Differentiate:

$$y = x \tan 2x$$

6. Differentiate:

$$y = \tan^4 x$$

7. Differentiate:

$$y = \frac{\operatorname{cosec} 2x}{x^2}$$

8. Differentiate:

$$y = \sec^3 x$$

9C (Part 1) dx/dy

1. Find the value of $\frac{dy}{dx}$ at the point (2,1) on the curve with equation $y^3 + y = x$

9G Differentiating Parametric Equations

1. Find the gradient at the point P where $t = 2$, on the curve given parametrically by:

$$x = t^3 + t, \quad y = t^2 + 1, \quad t \in \mathbb{R}$$

2. Find the equation of the normal at the point P , where $\theta = \frac{\pi}{6}$, to the curve with parametric equations $x = 3\sin\theta$ and $y = 5\cos\theta$.

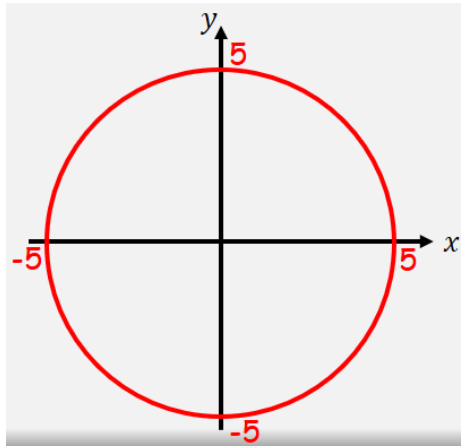
9H Implicit Differentiation

1. Differentiate the following equation implicitly:

$$y^3 = 3x^2$$

2. Below is a sketch of the circle with equation $x^2 + y^2 = 25$, $-5 \leq x \leq 5$, $-5 \leq y \leq 5$.

Find the gradient of the curve where $x = 4$



3. Find $\frac{dy}{dx}$ in terms of x and y when:

$$x^3 + x + y^3 + 3y = 6$$

4. Given that $4xy^2 + \frac{6x^2}{y} = 10$, find the value of $\frac{dy}{dx}$ at the point (1,1)

5. Find the value of $\frac{dy}{dx}$ at the point (1,1), when:

$$e^{2x} \ln y = x + y + 2$$

9I Concave & Convex Functions

1. Find the interval on which the function $f(x) = x^3 + 4x + 3$ is concave.

2. Show that the function $f(x) = e^{2x} + x^2$ is convex for all real values of x .

3. The curve C has equation $y = x^3 - 2x^2 - 4x + 5$

a) Show that C is concave on the interval $[-2,0]$ and convex on the interval $[1,3]$

b) Find the coordinates of the point of inflection

9J Chain Rule in Context

1. Given that the area of a circle $A \text{ cm}^2$ is related to its radius $r \text{ cm}$ by the formula $A = \pi r^2$, and that the rate of change of its radius in cm s^{-1} is given by $\frac{dr}{dt} = 5$, find $\frac{dA}{dt}$ when $r = 3$

2. The volume of a hemisphere $V \text{ cm}^3$ is related to its radius $r \text{ cm}$ by the formula $V = \frac{2}{3}\pi r^3$, and the total surface area $S \text{ cm}^2$ is given by the formula $S = 3\pi r^2$. Given that the rate of increase of volume, in $\text{cm}^3 \text{ s}^{-1}$, $\frac{dV}{dt} = 6$, find the rate of increase of the surface area, $\frac{dS}{dt}$.

3. In the decay of radioactive particles, the rate at which particles decay is proportional to the number of particles remaining. Write down a differential equation for the rate of change of the number of particles.

4. Newton's law of cooling states that the rate of loss of temperature of a body is proportional to the excess temperature of the body compared to its surroundings. Write an equation that expresses this law.

5. The head of a snowman of radius R cm loses volume by evaporation at a rate proportional to its surface area. Assuming that the head is spherical, that the volume of a sphere is given by $V = \frac{4}{3}\pi R^3$ cm³ and that the surface area is $A = 4\pi R^2$ cm², write down a differential equation for the rate of change of radius of the snowman's head.