**GCE A level Mathematics (9MA0) – Paper 3**

**Statistics & Mechanics**

**Summer 2018**

**Summer 2018 student-friendly mark scheme**

**Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn’t show follow-through marks (marks that are awarded despite errors being made) or special cases.**

**It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.**

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| **Guidance on the use of codes within this document** |
| M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect.  A1 – accuracy mark. This mark is generally given for a correct answer following correct working.  B1 – working mark. This mark is usually given when working and the answer cannot easily be separated.  Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer). |

**SECTION A: STATISTICS**

**Question 1 (Total 5 marks)**

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| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | *C* | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | | P(*C* = *c*) |  |  |  |  |  |  |  |  |  | | B1 | This mark is given for a correct set of values for *c* |
| B1 | This mark is given for correct probabilities consistent with a discrete uniform distribution |
| (b) | P(*C* < 4) = | B1 | This mark is given for using the correct model |
| (c) | Probability lower than expected suggest the model is not a good one | B1 | This mark is given for a correct statement |
| (d) | Could cover will vary from month to month and place to place, so Helen should use a non-uniform distribution | B1 | This mark is given for a correct statement |

**Question 2 (Total 7 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | H0: *ρ* = 0, H1: *ρ* < 0 | B1 | This mark is given for stating both hypotheses in terms of *ρ* |
| Significance level = 0.05, sample size = 8  Critical value = –0.6215 | M1 | This mark is given for a correct critical value found |
| *r* < –0.6215 so the result is significant and there is evidence of a negative correlation between *w* and *t* | A1 | This mark is given for a correct statement supported by correct values |
| (b) | As the temperature increases, people tend to spend more time outside and less time shopping | B1 | This mark is given for a correct possible reason |
| (c) | Since *r* is close to –1, it is consistent with Tessa’s suggestion | B1 | This mark is given for a correct comment |
| (d) | *t* will be the explanatory variable since sales are likely to depend on the temperature | B1 | This mark is given for a correct statement |
| (e) | Every degree rise in temperature leads to a drop in weekly sales figures of £171 | B1 | This mark is given for a correct interpretation |

**Question 3 (Total 11 marks)**

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| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | The probability of a dart hitting the target is constant | B1 | This mark is given for a correct statement |
| Each throw of a dart is independent | B1 | This mark is given for a correct statement |
| (b) | P(*H* ≥ 4) = 1 – P(*H* ≤ 3) = 1 – 0.9872  = 0.0128 | B1 | This mark is given for a correct answer only (may be found using a calculator) |
| (c) | P(*F* = 5) = 0.94 × 0.1 | M1 | This mark is given for finding a probability expression in the form (1 – *p*)4 × *p* |
| = 0.06561 | A1 | This mark is given for a correct answer only |
| (d) | |  |  |  |  |  | | --- | --- | --- | --- | --- | | *n* | 1 | 2 | … | 10 | | P(*F* = *n*) | 0.01 | 0.01 + α | … | 0.01 + 9α | | M1 | This mark is given for setting up the distribution of *F* |
| The sum of probabilities = 1 | M1 | This mark is given for finding the sum of probabilities |
| (2 × 0.01 + 9*α*) = 1 | A1 | This mark is given for a correct equation containing *α* |
| *α* = 9*α* = 0.18  *α* = 0.02 | A1 | This mark is given for the correct answer only |
| (e) | P(*F* = 5) = 0.01 + 4*α* = 0.01 + (4 × 0.02)  = 0.09 | B1 | This mark is given for the correct answer only |
| (f) | Peta’s model assumes the probability of hitting the target is constant  Thomas’s model assumes the probability of hitting the target increases with each attempt | B1 | This mark is given for a correct statement describing the differences in the models |

**Question 4 (Total 13 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | Convenience sampling  or  Opportunity sampling | B1 | This mark is given for a correct sampling method |
| (b) | Quota sampling | B1 | This mark is given for a correct sampling method |
| For example, take 4 people every ten minutes | B1 | This mark is given for a correct description of quota sampling |
| (c) | Census | B1 | This mark is given for a correct statement |
| (d) | 58 – 26 = 32 minutes | B1 | This mark is given for the correct answer only |
| (e) | *μ* =  =  = 43.5 minutes | B1 | This mark is given for finding the mean |
| *σ x* = =  = √236.7026 | M1 | This mark is given for a method to find an expression for the standard deviation |
| 15.4 minutes | A1 | This mark is given for the correct answer only |
| (f) | There are outliers in the data (or data is skew) which will affect the mean and the standard deviation; | B1 | This mark is given for a correct statement mentioning outliers or skewness |
| therefore use the median and the interquartile range | B1 | This mark is given for a correct statement to recommend use of the median and the interquartile range |
| (g) | The value of 20, the lower quartile and the outliers will not change; only the median and the upper quartile have changed | B1 | This mark is given for a correct statement |
| The two values have moved below 40 and therefore more than 50% below the median 40; the two values have moved below the upper quartile 58 and therefore more than 75% below 58 while the other values stay the same | M1 | This mark is given for a correct explanation that one of the values of the median *or* the upper quartile have decreased |
| The values of the median and the upper quartile have both decreased | A1 | This mark is given for a correct statement |

**Question 5 (Total 14 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | P(*L* > 16) = 0.69146 | B1 | This mark is given for the correct answer only (this could be found using a calculator) |
| (b) | P(*L* > 20 ⏐ *L* > 16) = | M1 | This mark is given for a first step to identifying a suitable conditional probability |
| = | A1 | This mark is given for finding the ratio of probabilities |
| = 0.44621 | A1 | This mark is given for the correct answer only |
| (0.44621)4 | M1 | This mark is given for a method to find the probability |
| = 0.0396 | A1 | This mark is given for the correct answer only |
| (c) | [P(*L* > 4)]2 × [P(*L* > 20 ⏐ *L* > 16)]2 | M1 | This mark is given for a correct first step to finding a solution |
| = (0.99976)2 × (0.44621)2 | A1 | This mark is given for evaluating the expression found |
| = 0.199 | A1 | This mark is given for the correct answer only |
| (d) | H0: *μ* = 18, H1: *μ* > 18 | B1 | This mark is given for stating both hypotheses in terms of *μ* |
| ~ N | M1 | This mark is given for selecting a suitable model |
| P(> 19.2) = P(*Z* > 1.3146) = 0.089856 | A1 | This mark is given for using the model correctly |
| 0.089856 > 0.05, so not significant | A1 | This mark is for a correct non-contextual conclusion |
| There is insufficient evidence to support Alice’s belief that the mean lifetime is more than 18 hours | A1 | This mark is given for a correct statement |

**SECTION B: MECHANICS**

**Question 6 (Total 6 marks)**

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| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
|  | **r** = **i** – 2*t* 2**j** + *c* | M1 | This mark is given for an attempt to integrate the expression for velocity with respect to time |
| A1 | This mark is given for a fully correct integration of the expression for velocity |
| When *t* = 1, **r** = 2**i** – 2**j** + *c*  When *t* = 4, **r** = 4**i** – 32**j** + *c* | M1 | This mark is given for substituting *t* = 1 and *t* = 4 into the expression for **r** |
| Point *A* has coordinates (2, –2)  Point *B* has coordinates (4, –32) | A1 | This mark is given for finding the coordinates of points *A* and *B* |
|  | M1 | This mark is given for a method to find the distance *AB* |
| = √904 | A1 | This mark is given for the correct answer only |

**Question 7 (Total 8 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | *R* + 40 sin *α* = 20*g* | M1 | This mark is given for a method to resolve vertically |
| A1 | This mark is given for a resolving vertically correctly |
| 40 cos *α* – *F* = 20*a* | M1 | This mark is given for a method to resolve horizontally |
| A1 | This mark is given for resolving horizontally correctly |
| *F* = 0.14*R* | B1 | This mark is given for finding an expression for *F* |
| *R* + 40 ×  = 20 × 9.8  *R* + 24 = 196  *R* = 172  40 cos *α* – *F* = 20*a*  40 ×  – (0.14 × 172) = 20*a*  7.92 = 20*a*  *a* = 0.396 m s–2 | A1 | This mark is given for a correct answer only |
| (b) | Pushing will increase the value of *R* which will increase the available (limiting) friction *F* | B1 | This mark is given for a correct statement |
| Increasing the friction *F* will decrease the acceleration *a* | B1 | This mark is given for a correct statement |

**Question 8 (Total 8 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | **r** = **u***t* + **a***t* 2  **r** = (7**i** – 10**j**), **u** = (2**i** – 3**j**)  ⇒ (7**i** – 10**j**) = 2(2**i** – 3**j**) + **a** × 22 | M1 | This mark is given for a method to find a vector expression for **a** |
| 7**i** – 10**j** = 4**i** – 6**j** + 2**a**  2**a** = 3**i** + 4**j**  **a** = (1.5**i** – 2**j**) | A1 | This mark is given for finding a correct vector expression for **a** |
| ⏐**a**⏐ = | M1 | This mark is given for a method to find the distance *AB* |
| = 2.5 m s–2 | A1 | This mark is given for a fully correct proof supported by working |
| (b) | **v** = **u + a***t*  **v** = (2**i** – 3**j**) + 2(1.5**i** – 2**j**) | M1 | This mark is given for a method to find a vector expression for **v** from *O* to *A* |
| = (5**i** – 7**j)** | A1 | This mark is given for finding a correct expression for **v** |
| **v** = **u + a***t*  **v** = (5**i** – 7**j**) + *t*(4**i** + 8.8**j**)  = (5 + 4*t*)**i** + (8.8*t* – 7)**j**  Since the direction of motion of *P* is north east,  (5 + 4*t*) **=** (8.8*t* – 7)  12 = 4.8*t* | M1 | This mark is given for a method to find a vector expression for **v** from *A* to *B* |
| *t* = 2.5 s | A1 | This mark is given for the correct answer only |

**Question 9 (Total 13 marks)**

| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| --- | --- | --- | --- |
| (a) | *T* × 2*a* sin *α* = *Mga* + 3*Mgx* | M1 | This mark is given for a method to take moments about *A* |
| A1 | This mark is given for correctly taking moments about *A* |
| *T* = =  =  = | A1 | This mark is given for a correct proof with supporting working |
| (b) | cos *α* = 2*Mg* | M1 | This mark is given for finding an equation for the magnitude f the horizontal component of the force |
| ×  = 2*Mg*  = 2*Mg*  (3*x* + *a*) = 3*a*  *x* = | A1 | This mark is given for the correct answer only |
| (c) | *Y* = 3*Mg* + *Mg* – sin *α* | M1 | This mark is given for a method to resolve vertically |
| A1 | This mark is given for correctly resolving vertically |
| *Y* = 4*Mg* –  ×  = | A1 | This mark is given for a correct expression for *Y* |
| tan *β* =  = | M1 | This mark is given for a method to find tan *β* |
| = | A1 | This mark is given for the correct answer only |
| (d) | ≤ 5*Mg* | M1 | This mark is given for a method to find an equation for the breaking point of the rope |
| *x* ≤ | A1 | This mark is given for a correct inequality |
| If the rope is not to break, it must not be further than  from *A* | B1 | This mark is given for a correct explanation |

**Question 10 (Total 15 marks)**

| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| --- | --- | --- | --- |
| (a) | 02 = (*U* sin *α*)2 – 2g(3 – 2) | M1 | This mark is given for a method to use the model and vertical motion |
| *U* 2 sin2 *α* = 2*g*  *U* 2 = | A1 | This mark is given for a complete proof supported by working |
| (b) | *s* = *ut* | M1 | This mark is given for a method to use the model and horizontal motion |
| 20 = *Ut* cos *α* | A1 |  |
| *s* = *ut* + *at*2 | M1 | This mark is given for a method to use the model and vertical motion |
| – = *Ut* sin *α* – *gt*2 | A1 | This mark is given for a correct answer |
| *t* =  – = *U*sin *α* –*g* | M1 | This mark is given for an expression after substituting for *t* |
| *U* 2 =  – =  – *g*  – =  – *g*  – = 20  – 100 | M1 | This mark is given for an expression after substituting for *U* 2 |
| – = 20 tan *α* – 100 tan2 *α* | A1 | This mark is given for a correct answer |
| (4 tan *α* – 1)(100 tan *α* + 5) = 0 | M1 | This mark is given for forming a quadratic equation to find a value of tan *α* |
| tan *α* =  α = 14° | A1 | This mark is given for finding a correct value of *α* (rejecting tan *α* = ­–0.02) |
| (c) | For example:  There will be air resistance  The ball will have dimensions  The ball will spin  There will be effects of the wind | B1 | This mark is given for a correct limitation stated |
| (d) | *U* = | M1 | This mark is given for a method to find the time *t* |
| 20 = *Ut* cos *α*  *t* =  *t*2 =  = tan2 *α*  *t*2 =  ×  ×  *t* 2 = | M1 | This mark is given for substituting for *U* |
| *t* =  = 1.13 | A1 | This mark is given for a correct answer only |